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Assignment-3

Ques1) Write the following statements in symbolic form.

(i) The crop will be destroyed if there is a flood.

p: There is a flood

q: The crop will be destroyed

in symbolic it will be written as

$p \rightarrow q$

(ii) If it rains then I will Not go to market.

p: it rains

q: I will go to market

$p \rightarrow \sim q$

(iii) I am in trouble if the work is Not finished on time.

p: The work is Finished on time

q: I am in trouble

$\sim p \rightarrow q$

(iv) The home team wins whenever it is raining

p: The home team wins

q: It is raining

$p \rightarrow q$

Ques2 Suppose the statement $p \vee q$ is false. Find the truth values of the following formulas:

(i) $\neg(p \wedge q) \rightarrow q$;
(ii) $\neg q \rightarrow p \wedge q$;
(iii) $\neg p \rightarrow q \longleftrightarrow p \rightarrow \neg q$.

($p \vee q$ is False) given

(i)

p	q	$p \wedge q$	$\sim p \wedge q$	$(\sim p \wedge q) \rightarrow q$
F	F	F	T	F

(ii)

p	q	$\sim q$	$p \wedge q$	$\sim q \rightarrow p \wedge q$
F	F	T	F	F

(iii) $\sim p \rightarrow q \longleftrightarrow p \rightarrow \sim q$

p	q	$\sim p$	$\sim q$	$\sim p \rightarrow q$	$p \rightarrow \sim q$	$\sim p \rightarrow q \leftrightarrow p \rightarrow \sim$
F	F	T	T	F	T	F

Ques3 Truth table of $(p \rightarrow \sim q) \rightarrow \sim p$

p	q	$\sim p$	$\sim q$	$p \rightarrow \sim q$	$(p \rightarrow \sim q) \rightarrow \sim p$
T	T	F	F	F	T
T	F	F	T	T	F
F	T	T	F	T	T
F	F	T	T	T	T

(ii) $p \leftrightarrow (\sim p \vee \sim q)$

q	p	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$p \leftrightarrow \sim p \vee \sim q$
T	T	F	F	F	F
F	T	F	T	T	T
T	F	T	F	T	F
F	F	T	T	T	F

Ans4 Tautology-: A compound Proposition that is always true For all Possible truth values of its variables is called Tautology.

Contradiction-: A compound Proposition that is always false for all Possible truth values of its variables is called contradiction.

Contigency-: A proposition that is neither a Tautology Nor a contradiction is called contigency.

Satisfiability-: A compound proposition is satisfiable if it is true for some assignment of truth values to its variables.

(i) $(p \vee q) \wedge (\sim p \vee r) \longrightarrow (q \vee r)$

Let $X = (p \vee q) \wedge (\sim p \vee r) \longrightarrow q \vee r$

p	q	r	~p	p∨q	~p∨r	q∨r	(p∨q)∧(~p∨r)	X
T	T	T	F	T	T	T	T	T
T	T	F	F	T	F	T	F	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	F	T	T	F	T
F	F	F	T	F	T	F	F	T

(Hence it is a tautology)

ues5 Use algebric method to determine which of the following is a tautology or a contradiction.

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(i) $\sim(q\to r)\wedge r\wedge(p\to q)$
(ii) $((p\vee q)\wedge(p\to r)\wedge(q\to r))\to r$

Soln (i) given,

$\Rightarrow \sim(q\to r)\wedge r\wedge(p\to q)$

$\Rightarrow \sim(\sim q\vee r)\wedge r\wedge(\sim p\vee q)$

$\Rightarrow \sim(\sim q)\vee\sim r\wedge r\wedge(\sim p\vee q)$

$\Rightarrow q\vee\sim r\wedge r\wedge(\sim p\vee q)$

$\Rightarrow q\vee F\wedge(\sim p\vee q)$

$\Rightarrow F\wedge(\sim p\vee q)$

$\Rightarrow F\wedge(p\to q)$

$\Rightarrow F$

$\therefore$ the given expression / statement will be a <u>contradiction</u>.

(ii) given,

$((p\vee q)\wedge(p\to r)\wedge(q\to r)]\to r$

$\Rightarrow [(p\vee q)\wedge(\sim p\vee r)\wedge(\sim q\vee r)]\to r$

$\Rightarrow \neg((p\vee q)\wedge(\sim p\vee r)\wedge(\sim q\vee r)\vee r$

$\Rightarrow \sim((p\vee q)\wedge r(\sim p\vee\sim q)\vee r)\vee r$

$\Rightarrow (\sim p\vee\sim q)\wedge(p\vee q)\vee\sim r]\vee r$

$\Rightarrow [F\vee\sim r]\vee r \Rightarrow T\vee r$

$= T$

Hence the given statement will be a <u>tautology</u>

Ans 7) (i) $p \to q \equiv \sim p \vee q \equiv \sim q \to \sim p$

p	q	$p \to q$	$\sim p$	$\sim p \vee q$	$\sim q$	$\sim q \to \sim p$
T	T	T	F	T	F	T
T	F	F	F	F	T	F
F	T	T	T	T	F	T
F	F	T	T	T	T	T

(ii) $p \Longleftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p) \equiv (p \vee q) \vee (\sim p \wedge \sim q)$

Let $(p \vee q \vee \sim p \wedge \sim q) = X$

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim p \wedge \sim q$	X
T	T	F	F	T	F	T
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	T

Ans 8 $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

$(p \to r) \wedge (q \to r) = (\sim p \vee r) \wedge (\sim q \vee r)$

$= (\sim p \wedge \sim q) \vee r$

$= \sim(p \vee q) \vee r$ (De-morgan)

$= (p \vee q) \to r$ (Implication as disjunction

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(ii) $((p \wedge q \wedge a) \to r) \wedge (a \to (p \vee q \vee r)) \equiv (a \wedge (p \leftrightarrow q) \to r$

$((p \wedge q \wedge a) \to r) \wedge (a \to (p \vee q \vee r))$

$\equiv (\sim (p \wedge q \wedge a) \vee r) \wedge (\sim a \vee (p \vee q \vee r))$

$\equiv ((\sim p \vee \sim q \vee \sim a) \vee r) \wedge (\sim a \vee p \vee q) \vee r)$

(de-morgan)

$\equiv ((\sim p \vee \sim q \vee \sim a) \vee r) \wedge ((p \vee q \vee \sim a) \vee r)$

(commutative)

$\equiv ((\sim p \vee \sim q \vee \sim a) \wedge (p \vee q \vee \sim a)) \vee r$

(distributive)

$\equiv (((\sim p \vee \sim q) \wedge (p \vee q)) \vee \sim a) \vee r$

(distributive)

$\equiv (\sim [(p \wedge q) \vee (\sim p \wedge \sim q)] \vee \sim a) \vee r$

(de-morgan)

$\equiv \sim ([(p \wedge q) \vee (\sim p \wedge \sim q) \wedge a) \vee r$

(de-morgan)

$\equiv \sim ((p \leftrightarrow q) \wedge a) \vee r$

$\equiv ((p \leftrightarrow q) \wedge a) \to r$ (Implication as disjunction)

ques 9) $(p \to q) \wedge p \Rightarrow q$

we have,

$(p \to q) \wedge p \to q \equiv (\sim p \vee q) \wedge p \to q$

$\equiv ((\sim p \wedge p) \vee (q \wedge p)) \to q$

$\equiv (F \vee (q \wedge p)) \to q$

$\equiv \sim (q \wedge p) \to q$

$\equiv (\sim q \vee \sim p) \vee q$

$\equiv (\sim q \vee q) \vee \sim p$
$\equiv T \vee \sim p$
$\equiv T$

(ii) $((p \to q) \wedge \sim q \Rightarrow \sim p$

$(\sim p \vee q) \wedge \sim q \to \sim p$ (Impli. as disjunction)
$\equiv ((\sim p \wedge \sim q) \vee (q \wedge \sim q)) \to \sim p$ (distributive)
$\equiv (\sim (p \vee q) \vee F) \to \sim p$ (De-morgan)
$\equiv \sim (p \vee q) \to \sim p$ (Identity-Law)
$\equiv p \vee q \vee \sim p$
$\equiv (p \vee \sim p) \vee q$ (commutative)
$\equiv T \vee q$ (compliment)
$\equiv T$ (dominance)

(iii) ~~$(p \to q) \wedge (q \to r) \Rightarrow (p \to r)$~~

$(p \vee q) \wedge \sim p \Rightarrow q$
$((p \wedge \sim q) \vee (q \wedge \sim q)) \to p$ (distributive)
$\equiv ((p \wedge \sim q) \vee F) \to p$ (compliment law)
$\equiv (p \wedge \sim q) \to p$ (Identity-Law)
$\equiv \sim (p \wedge \sim q) \vee p$ (Impli. as disjunction)
$\equiv (\sim p \vee q) \vee p$ (de-morgan)
$\equiv (\sim p \vee p) \vee q$ (commutative)
$\equiv T \vee q$ (Identity Law)
$\equiv T$ (dominance)

Ques10) Rule of Inference:-

(1) Modus Ponen:- If the statement p and $p \to q$ are accepted as true then q must be true.

$$\begin{array}{l} p \to q \\ \underline{p} \\ \therefore q \end{array}$$ is valid

(2) Modus Tollen:- if statement $p \to q$ and $\sim q$ are true then $\sim p$ must be true.

$$\begin{array}{l} p \to q \\ \underline{\sim q} \\ \therefore \sim p \end{array}$$ is valid

(3) Hypothetical syllogism:- whenever the two implications $p \to q$ and $q \to r$ accepted as true then implication $p \to r$ is also true.

$$\begin{array}{l} p \to q \\ \underline{q \to r} \\ \therefore p \to r \end{array}$$ is true

(4) Disjunctive syllogism-: an argument of the form.

$$\begin{array}{l} p \vee q \\ \sim q \\ \hline \therefore p \end{array}$$ is valid

(5) Addition-: An argument of the form

$$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}$$ is valid.

(6) Simplification-: $\begin{array}{l} p \wedge q \\ \hline \therefore p \end{array}$ or $\begin{array}{l} p \wedge q \\ \hline \therefore q \end{array}$

(7) conjunction-: If p and q are true then $p \wedge q$ is also true.

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

(8) constructive dilemma-:

$$\begin{array}{l} p \to q \wedge (r \to s) \\ p \vee r \\ \hline \therefore p \vee s \end{array}$$

(9) destructive dilemma-:

$$\begin{array}{l} p \to q \wedge (r \to s) \\ \sim q \vee \sim s \\ \hline \therefore \sim p \vee \sim r \end{array}$$

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Ques11 **Consistent Premises-:** A set of premises $P_1, P_2 \ldots P_n$ are said to be consistent if their conjunction $P_1 \wedge P_2 \ldots P_n$ has truth value T in atleast one possible case of their variable.

Inconsistent Premises-: A set of premises $P_1 P_2 \ldots P_n$ are said to be Inconsistent if $P_1 \wedge P_2 \ldots \wedge P_n \rightarrow F$ i.e they lead to contradiction.

Ques12 $r \rightarrow \sim q,\ r \vee s,\ s \rightarrow \sim q,\ p \rightarrow q$ imply $\sim p$

1) $p \rightarrow q$ (Rule P)
2) $r \rightarrow \sim q$ (Rule P)
3) $q \rightarrow \sim r$ (Rule T on 2)
4) $p \rightarrow \sim r$ (H.S)
5) $s \rightarrow \sim q$ (Rule P)
6) $q \rightarrow \sim s$ (Rule T on 5)
7) $p \rightarrow \sim s$ (H.S) (1)(6)
8) $r \rightarrow \sim p$ (Rule T)
9) $s \rightarrow \sim p$ (Rule T)
10) $(r \rightarrow \sim p) \wedge (s \rightarrow \sim p)$ (Conjunction) (8)(9)
11) $r \vee s$ (Rule P)
12) $\sim p \vee \sim p$ (10)(11) (Constructive)
13) $\sim p$ Idempotent

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Ques13)

E: Nermala misses many classes
S: Nermala fails high school
A: Nermala reads lot OF books
H: Nermala is uneducated.

1) $E \to S$ (Rule P)
2) $S \to H$ (Rule P)
3) $E \to H$ (Rule 1 on 1) 2)
4) $A \to \sim H$ (Rule P)
5) $H \to \sim A$ (Rule T)
6) $E \to \sim A$ (Rule T) (4)
7) $\sim E \to \sim A$ (Rule T (3)(5)
8) $\sim (E \land A)$ (Rule T (6)
9) $\sim (E \land A)$ Rule T (7)
10) $E \land A$ (Rule P)
11) $(E \land A) \land \sim (E \land A)$ (Rule T) (8)(9)
$\equiv F$

Therefore set of premises are inconsistent

Ques14)

a: It rains
b: It is foggy
c: The sailing race will be held
d: The lifesaving demonstration will go on
e: The tropy will be awarded.

$(\sim a \vee \sim b) \to (c \wedge d)$, $c \to e$ and $\sim e$

Premise	Argument
$\sim e$	Rule P
$c \to \sim e$	Rule P
$\sim c$	Modus Tollen.
$(\sim a \vee \sim b) \to (c \wedge d)$	(Rule P)
$\sim (c \wedge d) \to \sim(\sim a \vee \sim b)$	(Rule T)
$(\sim c \vee \sim d) \to (a \vee b)$	(De-morgan)
$\sim c \vee \sim d$	(Addition using)
$a \wedge b$	
r	(simplification)

Ques 15) Justify that the premise" It is Not sunny - - -

p: It is sunny this afternoon
q: It is colder than yesterday
r: we will go to swimming
s: we will take a canoe trip
t: we will be home by sunset

$\sim p \wedge q$, $r \to p$, $\sim r \to s$ and $s \to t$

1) $\sim p \wedge q$ (Rule P)

2) $\sim p$ (1)

3) $r \rightarrow p$ (Rule-P)

4) $\sim r$ (Modus Tollen) on (2)(3)

5) $\sim r \rightarrow s$ (Rule P)

6) s (Modus Tollen)

7) $s \rightarrow t$ (Rule P)

8) t (Modus Ponen) (6)(7)

Ques 6 (i) Not all birds can fly

$B(x)$: x is a Bird

$F(x)$: x can fly

So, $\sim [\forall x [B(x) \rightarrow F(x)]] \rightarrow \exists x [B(x) \wedge \sim F(x)]$

(ii) Everybody loves somebody.

Let $L(x, y)$: x loves y

So, $\forall x \exists y \, L(x, y)$

(iii) All students need Financial aids.

Let $S(x)$: x is a student

$F(x)$: x needs financial aids

So, $\forall x [S(x) \rightarrow F(x)]$

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(iv) All integers are either odd or even

Let $E(x)$: x is even
$O(x)$: x is odd
So, $\forall x [E(x) \vee O(x)]$

(v) If a person is female and is a parent then this person is someone's mother.

Let $F(x)$: x is female
$P(x)$: x is parent
$M(x, y)$: x is the mother of y
So, $\forall x [F(x) \wedge P(x)] \rightarrow \exists y\, M(x, y)$

(vi) All flowers are beautiful

$F(x)$: x is a flower
$B(x)$: x is beautiful

So, $\forall x [F(x) \rightarrow B(x)]$

Ans17) Universal modus ponens-: The rule of universal ~~initiate~~ intantiation can be combined with modus ponen to obtain the rule called U modus ponen.

$$\begin{array}{l} \forall x, (P(x) \rightarrow Q(x)) \\ P(a) \\ \hline \therefore Q(a) \end{array}$$

Universal modus tollens-:

$$\begin{array}{l} \forall x (P(x) \rightarrow Q(x)) \\ \sim Q(a) \\ \hline \therefore \sim P(a) \end{array}$$

a is some element of universe

(Rule US) ⇒ $\forall x\, P(x)$ is true iff $P(x)$ is true for every x in the universe of discourse the impliplication $x P(x) \Rightarrow P(y)$ holds.

(Rule ES): If $(\exists x) P(x)$ is true then $P(x)$ is true for some x in the universe and we have the implication.

$$(\exists x) P(x) \Rightarrow P(a)$$

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Rule of Generalization -: Rule UG when we have $P(x)$ is true for all x it follows that $\forall x P(x)$ is true i.e

$$P(x) \Rightarrow \forall y P(y) \text{ holds}$$

Rule EG -: If $P(x)$ is true for at least one subject $x=a$ in the universe of discourse then we have the conclusion $P(a) \Rightarrow (\exists x) P(x)$ this is known as Rule EG.

Ques 18) Show that -:

(i) $\forall x[P(x) \vee Q(x)] \Rightarrow x P x \vee \exists Q(x)$

$\forall x [P(x) \vee Q(x)]$ —— (i)

(1) $\sim[\forall x P(x) \vee \exists x Q(x)]$

(2) $[\sim \forall x P(x)] \wedge \sim [\exists x Q(x)]$

(De-morgan)

(3) $\exists x (\sim P(x)) \wedge \forall x [-Q(x)]$

(Negation)

(4) $\exists x (\sim P(x))$

(5) $\forall x [\sim Q(x))]$

(Simplification)

(6) $\exists \sim P(a)$

(7) $\sim Q(a)$

(Essential Instantaneous)

(8) $\sim P(a) \wedge \sim Q(a)$ (Conjuction)
(9) $\sim [P(a) \vee Q(a)]$ (De-morgan)
(10) F (on taking conjuction of (9) and i)

(ii) $(x) P(x) \wedge \exists Q(x) \Rightarrow \exists [P(x) \wedge Q(x)]$

(1) $\exists x [P(x) \wedge Q(x)]$ (Premise)
(2) $P(a) \wedge Q(y)$ (essential specification)
(3) $P(a)$ } (simplification)
(4) $Q(a)$
(5) $\exists x P(x)$ } (essential generalization)
(6) $\exists x Q(x)$

(7) $\exists x P(x) \wedge \exists x Q(x)$ [5,6 conjunction]

Ans20 (i) For every no there is a Number greater than that Numbers.

Let x and $y \in \mathbb{Z}^+$
For all x; there exists a y such that y is greater than x

Let $P(x,y)$: y is greater than x
The given statement is
$(\forall x)(\exists y) P(x,y)$

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(ii) Sum of Every two integer is an integer

x: a is an integer
y: b is an integer
$P(x,y)$: $a+b$ is an integer

Hence $\forall x \forall y\ P(x,y)$

(iii) Not Every man is perfect

Let $H(x)$: x is a man
$G(x)$: x is Not perfect

Hence,
$\exists x\,[H(x) \land G(x)]$

(iv) There is No student in class who known both spanish and German.

$S(x)$: x knows spanish
$G(x)$: x knows German
$\forall x\,[\sim G(x) \land S(x)]$